



A dynamic model for the efficiency optimization of an oscillatory low grade heat engine

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ABSTRACT

A simple approach is presented for the modeling of complex oscillatory thermal-fluid systems capable of converting low grade heat into useful work. This approach is applied to the NIFTE, a novel low temperature difference heat utilization technology currently under development. Starting from a first-order linear dynamic model of the NIFTE that consists of a network of interconnected spatially lumped components, the effects of various device parameters (geometric and other) on the thermodynamic efficiencies of the device are investigated parametrically. Critical components are highlighted that require careful design for the optimization of the device, namely the feedback valve, the power cylinder, the adiabatic volume and the thermal resistance in the heat exchangers. An efficient NIFTE design would feature a lower feedback valve resistance, with a shorter connection length and larger connection diameter; a smaller diameter but taller power cylinder; a larger (time-mean) combined vapor volume at the top part of the device; as well as improved heat transfer behavior (i.e. reduced thermal resistance) in the hot and cold heat exchanger blocks. These modifications have the potential of increasing the relevant form of the second law efficiency of the device by 50% points, corresponding to a 3.8% point increase in thermal efficiency.

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1. Introduction

With ever-increasing environmental concerns, including that of climate change, but also of energy security in the light of finite resources of common fossil fuels, it is becoming increasingly important to consider alternative clean and sustainable energy solutions. The efficiency and power density of appropriate thermodynamic systems are key quantities on which their usefulness and wider application rely. This is especially true for devices that are capable of operating with marginal energy sources and close to ambient temperatures. In this paper we present a modeling framework based on thermodynamic, heat transfer and fluid-mechanical principles for the early stage development of oscillatory low grade heat (e.g. solar energy or waste heat) utilization systems. We demonstrate the approach by applying it to a promising new low grade heat utilization technology that has attracted attention recently as a result of potential reliability advantages as well as reduced capital and operating costs, on account of its few moving parts relative to mechanical heat engines. Specifically, we consider the effects of various system

parameters on a number of key efficiency definitions that can be formulated in order to describe the performance of the system.

The “Non-Inertive-Feedback Thermofluidic Engine” (NIFTE), as proposed in Refs. [1–3], can be described as a “two-phase unsteady heat engine” in which persistent and reliable thermodynamic (pressure, temperature, etc.) oscillations are generated and sustained by *steady* external temperature differences. These desirable oscillations are driven by and give rise to heat and fluid flows, which involve the evaporation (boiling) and condensation of the working fluid. Hence, the NIFTE can also be considered a two-phase realization of a class of devices known as “thermofluidic oscillators”, which includes thermoacoustic engines [4–7], dry free-liquid-piston (Fluidyne) engines [8–11], free-piston Stirling engines [12–14], pulsejets and pulse-tubes [15–19]. In common with other two-phase thermofluidic oscillators, the NIFTE is particularly well suited to the conversion of low grade heat. It can be used to produce useful (hydraulic) work for fluid pumping, heating and/or cooling and niche power generation applications; in a variety of solar thermal and energy recovery settings, and in combined heat and power-pumping (CHP) schemes. The NIFTE has been demonstrated as being capable of operating across temperature differences down to 30 K between the heat source and sink.

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Nomenclature		Dimensionless	
C	Electrical capacitance equivalent ($\text{m}^4 \text{s}^2 \text{kg}^{-1}$)	G	Linear system gain (–)
$E (\rightarrow P)$	Electrical potential/voltage equivalent (Pa)	Re	Reynolds number (–)
$I (\rightarrow U)$	Electrical current equivalent ($\text{m}^3 \text{s}^{-1}$)	Wo	Womersley parameter (–)
L	Electrical inductance equivalent (kg m^{-4})	c_d	Dimensionless friction factor/drag coefficient (–)
P	Pressure (Pa)	$\gamma = c_p/c_v$	Ratio of specific heat capacities (–)
Q	Heat flow (J)	η	Thermodynamic efficiency (–)
R	Electrical resistance ($\text{kg m}^{-4} \text{s}^{-1}$)	Operations	
S	Entropy (J K^{-1})	$\langle (\cdot) \rangle$	Averaged, as stated
T	Temperature ($^{\circ}\text{C}$ or K)	$\overline{(\cdot)}$	Time-mean average
$U = \dot{V}$	Volumetric flow rate ($\text{m}^3 \text{s}^{-1}$)	$\overline{(\cdot)} = \mathcal{L}\{\cdot\}$	Laplace transform of
V	Volume (m^3)	$(\cdot)' = (\cdot) - \overline{(\cdot)}$	Temporal fluctuation of a quantity around its mean
W	Work done by a thermodynamic system (J)	$(\dot{\cdot}) = d(\cdot)/dt$	Rate of change, or flow rate of
X	Exergy (J)	$(\cdot)^*$	Value of a parameter relative to its nominal or maximum investigated value
Z	Electrical impedance equivalent ($\text{kg m}^{-4} \text{s}^{-1}$)	$\angle \{\cdot, \cdot\}$	Phase angle between the stated variables
c_p	Specific isobaric heat capacity at constant pressure ($\text{J kg}^{-1} \text{K}^{-1}$)	$\Delta(\cdot)$	Change/difference in
c_v	Specific isochoric heat capacity at constant volume ($\text{J kg}^{-1} \text{K}^{-1}$)	Other	
d	Diameter (m)	ad	Adiabatic process or adiabatic vapor chamber
$f = 1/\tau$	Frequency of a cycle or device (Hz)	ao	Mean value during an adiabatic process in the adiabatic vapor chamber
$g = 9.81$	Gravitational acceleration (m s^{-2})	c	Cold, or heat sink
h	Enthalpy per unit mass of working fluid, or specific enthalpy (J kg^{-1})	d	Displacer cylinder
k	Thermal conductance or inverse of the thermal resistance (W K^{-1})	dev	Device, or overall/global to the complete fluid pumping device
l	Length (m)	ex	Exergetic
m	Mass (kg)	f	Feedback, or feedback connection and valve
s	Entropy per unit mass of working fluid, or specific enthalpy ($\text{J kg}^{-1} \text{K}^{-1}$)	fg	Change over an evaporative phase change process
u	Bulk flow velocity averaged over flow cross-section (m s^{-1})	h	Hot, or heat source
$v = 1/\rho$	Volume per unit mass of working fluid, or specific volume ($\text{m}^3 \text{kg}^{-1}$)	in	Into the thermodynamic system over a cycle
Greek		l	Load
μ	Dynamic or absolute viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)	o	Mean/time-averaged value
ρ	Density (kg m^{-3})	out	Out of the thermodynamic system over a cycle
$\tau = 1/f$	Cycle period (s)	p	Power cylinder
$\omega = 2\pi f$	Angular frequency (rad s^{-1})	s	Common designation for heat source and heat sink
		sat	For a fluid in the saturation region
		th	Thermal, or thermal domain involving the heat exchangers
		wfl	Working fluid, or specific to the working fluid thermodynamic cycle

Thermofluidic oscillators have many dynamic similarities with analog electronic oscillator circuits, and thus, electrical analogies have been used to predict approximate stability/instability criteria and to estimate first-order heat and work flows, and efficiencies. The analogies, used extensively by Backhaus and Swift [4] in the analysis and development of thermoacoustic engines, were extended to include a description of exergy (or availability) flows and to allow for exergy losses due to irreversible heat transfer in order to model the NIFTE by Smith [1]. In this work a “conventional” exergetic efficiency [20] was introduced as the second law efficiency most appropriate to thermodynamic models described by electrical analogies in which the heat source and heat sink temperatures are represented as an externally imposed linear temperature gradient rather than separate constants. This exergetic efficiency, defined below in Section 2.4, is independent of absolute temperatures as well as temperature differences (and amplitudes). It is therefore conveniently invariant to changes in operating

conditions and was proposed as a general figure-of-merit for the characterization of the thermodynamic reversibility of the engine, and by extension, its performance.

The electrical analogy model of Smith [1–3] showed good agreement with experimental trends obtained from measurements on an actual NIFTE prototype over a certain range of conditions and prototype design parameters, though it also showed non-negligible deviations in some cases [2,3]. Considerable effort is placed in these references to explain the underlying reasons for these deviations, including for example the existence of shuttle (irreversible heat transfer) loss mechanisms in non-heat exchanging (or, nominally adiabatic) components, and the presence of fluid (liquid) inertia in the real NIFTE. Unfortunately, the original model was not extended to include such effects, and in particular, although mentioned in this early work the effect of inertia was not studied.

Yet, any real embodiment of such a system will encompass inertial effects to some extent due to the finite mass of any

contained liquids. A dependence on inertia in order to allow the system sustained pressure oscillations with large amplitudes, which is vital for the achievement of high power densities, implies a requirement for bulky tuning lines and resonators that the designer would like to avoid. An understanding of the need for and of the effects of finite inertia is crucial for the design of the next generation of these novel and complex systems, both in terms of enhanced efficiency and performance. The effects of inertia were incorporated into the original “non-inertive” electrical analogy model of Smith [1], by Solanki et al. [21]. The current work uses this latter “inertive” model as a starting point.

In Solanki et al. [21] the authors studied the effect of introducing (and parametrically varying the degree of) inertia on key NIFTE performance indicators, such as the minimum temperature gradient between the heat source and sink necessary for operation (continuous oscillations) and the resulting oscillation frequency. The revised inertive model resulted in a much improved prediction of these indicators relative to the original non-inertive equivalent, over a wider range of values. This suggests that the inclusion of inertia is indeed essential for the correct capture of the observed behavior of the NIFTE. The inertive model was then used to investigate the expected efficiency of the NIFTE for a typical and realistic set of system properties and design parameters.

However, this previous study focused only on the exergetic (second law) efficiency of the engine and did not present results for its thermal (first law) efficiency. In addition we note that, in general, the real NIFTE is an energy producing device that can have a different (indeed, reduced) global efficiency from the efficiency of its enclosed thermodynamic cycle, owing to processes of parasitic dissipation that are external to the working fluid cycle, but still internal to the overall device. In fact, the NIFTE depends on a throttle valve (referred to as the “feedback valve”) to create a phase shift between the heat flow and the power stroke. The existence of this phase shift results in a parasitic fluid-mechanical viscous loss that leads to a difference between the efficiency of the thermodynamic cycle undergone by the working fluid, and the overall or global efficiency of the complete device. In Smith [1–3] the effect of this component on the exergetic efficiency was assumed negligible. In Solanki et al. [20], it was included throughout. However, only its effect on the overall exergetic efficiency of the whole engine was considered. Whilst this provides some useful insights, increases in exergetic efficiency may lead to decreases in overall thermal efficiency, since decreases in thermal dissipation and parasitic losses are often accompanied by a reduction in the temperature difference between heat addition and heat rejection. Hence, a more comprehensive approach is required for a complete understanding of the NIFTE.

In this paper we use the extended inertive model of the NIFTE [21] to investigate the effect of the fluid-mechanical parasitic losses both on the exergetic efficiencies and on the thermal efficiencies of the NIFTE pump. We refer to efficiencies in which the parasitic fluid-mechanical losses are ignored as “working fluid” efficiencies, and efficiencies in which they are included as “device” efficiencies. Therefore, with regards to the NIFTE, one needs to distinguish between four measures of efficiency: thermal and exergetic overall/global device efficiencies; and thermal and exergetic working fluid efficiencies. Solanki et al. [21] restricted their attention to the exergetic overall device efficiency, while in Refs. [1–3] the parasitic losses were assumed to be negligible altogether (i.e. no distinction was made between device and system efficiencies). Here, we investigate the consequences of this assumption and show that it is not valid in certain regions of the parameter space. We use the results of this process to construct a framework for including parasitic losses in future thermofluidic oscillator models, in order to gain a more complete understanding of the thermodynamic performance potential of the NIFTE.

2. Methods

2.1. Problem formulation: heat, power, exergy and efficiency definitions

Consider an arbitrary, externally heated work-producing device, within which power is generated by a working fluid that undergoes a thermodynamic cycle with frequency f . Rather than dealing with thermodynamic states we will deal with flow rates of thermodynamic properties (e.g. “entropy flow rate” $\dot{S}$), so that irreversible processes may be quantified legitimately as increases in entropy flow rate. Lower case letters are used to indicate specific quantities (per unit mass of working fluid), such that for instance $\dot{s}$ is the specific entropy flow rate.

The temperature of the working fluid of the device that undergoes this “main cycle” can be written as a decomposition $T = T_o + T'$, where T' are fluctuations about the time-averaged value T_o . By definition the time-average of the fluctuations is zero $\overline{T'} = 0$. The working fluid is heated by an external source and cooled by an external sink, through contact with appropriate heat exchangers. The source and sink on the other side of the heat exchangers undergo separate processes. Let us denote by T_h the temperature of the source from which the main cycle obtains heat, and by T_c the temperature of the sink to which it rejects heat. Assuming that the main cycle is heated and cooled by exclusively irreversible heat transfer, i.e. pure Fourier conduction (diffusion) and Newtonian convection (surface) phenomena, T_h and T_c are bounded by lines of constant heat flow rate $\dot{Q}$ that set a limit to the extent of the main cycle, wherein $T = \dot{Q}/\dot{S}$. In addition, let $\Delta S = \int_{\dot{S}>0} \dot{S} dt > 0$ be the

entropy change during the process of heat addition ($\dot{S} > 0$), such that the entropy change during heat rejection ($\dot{S} < 0$) is equal to this in magnitude but opposite in sign ($\Delta S < 0$); and let V be the volume of the working fluid, such that $U = \dot{V}$ is the volumetric displacement flow rate responsible for power generation.

Then, based on an arbitrary datum state (e.g. chose the time-averaged T_o , P_o), the rate of heat input to the main cycle $\dot{Q}_{in}$, the net power produced $\dot{W}$, and the “net exergy (or equivalently availability) flow rate” $\dot{X}$ are:

$$\dot{Q}_{in} = \int_{\dot{S}, \Delta S > 0} T(\dot{S}) d\dot{S} = f T_o \Delta S + \int_{\dot{S}, \Delta S > 0} T'(\dot{S}) d\dot{S} \quad (1)$$

$$\dot{W} = \oint P(\dot{V}) d\dot{V} = \oint P(U) dU = \dot{Q} = \dot{Q}_{in} - \dot{Q}_{out} = \oint T(\dot{S}) d\dot{S} \quad (2)$$

$$\begin{aligned} \dot{X}_{in} &= \int_{\dot{S}, \Delta S > 0} [T_h(\dot{S}) - T_o] d\dot{S}; & \dot{X}_{out} &= \int_{\dot{S}, \Delta S < 0} [T_c(\dot{S}) - T_o] d\dot{S}; \\ \dot{X} &= \dot{X}_{in} - \dot{X}_{out} = \oint T_s(\dot{S}) d\dot{S} \end{aligned} \quad (3)$$

where T_s is used to denote either T_h or T_c during the sequential heating ($\dot{S} > 0$; $\Delta S > 0$) and cooling ($\dot{S} < 0$; $\Delta S < 0$) parts of the cycle. In addition, we define the equivalent rate of heat gained from the heat source $\dot{Q}_h$ external to the main cycle as,

$$\dot{Q}_h = \int_{\dot{S}, \Delta S > 0} T_h(\dot{S}) d\dot{S} = f T_o \Delta S + \int_{\dot{S}, \Delta S > 0} T'_h(\dot{S}) d\dot{S} \quad (4)$$

which from Eq. (3) is actually the thermal exergy input $\dot{X}_{in}$ plus the constant (time-mean component) $f T_o \Delta S$ supplied by the heat source during the heat addition stage.

From Eq. (2) we can see that the power produced by the device $\dot{W} = \dot{Q}$ may be described by the area inside a T - S diagram that

represents the internally reversible part of the main thermodynamic cycle, multiplied by the frequency of the cycle f . This area is enclosed by a larger area that, when multiplied by frequency f , represents the net exergy flow rate $\dot{X}$ of the full irreversible cycle, with the difference between these areas representing power loss through irreversible processes. Hence, $\dot{X}$ is the maximum power available to the cycle, given the external boundary conditions. It is clear from Eqs. (2) and (3) that changes in T_o will have no effect on $\dot{W}$ or $\dot{X}$. Conversely, from Eqs. (1) and (4) we observe that both $\dot{Q}_{in}$ and $\dot{Q}_h$ do indeed depend on T_o , and consequently on the magnitude of relative to T_o .

Next we consider two classes of efficiency: (1) the thermal efficiency η_{th} , defined as $\eta_{th} = \dot{W}/\dot{Q}_h$; and (2) the exergetic efficiency η_{ex} , defined as $\eta_{ex} = \dot{W}/\dot{X}$. In earlier models of the NIFTE the parasitic fluid-mechanical viscous dissipation in the feedback valve was assumed negligible [1–3]. We refer to efficiencies based on this assumption as (thermal or exergetic) “working fluid” efficiencies η_{wfl} and efficiencies for which this fluid-mechanical dissipation is accounted for as (thermal or exergetic) “device” efficiencies η_{dev} . In this paper we consider the effect of various device parameters on these efficiencies, with reference to the NIFTE which is introduced in more detail below.

2.2. Non-Inertive-Feedback Thermofluidic Engine (NIFTE)

A schematic of the NIFTE can be seen in Fig. 1. Details of the construction and operation of the NIFTE can be found elsewhere [3,21], but briefly the NIFTE comprises two vertical cylinders (1,2), connected at the top (3) and bottom (4) with horizontal pipes. The whole interconnected volume contained within these main components (chambers and tubes) is filled with a single working fluid that exists in both the vapor (white) and the liquid (gray) phase at the same time. The liquid and vapor volumes are separated by two vapor–liquid interfaces denoted in Fig. 1 by levels (6) and (7), the first of which is inside the power cylinder on the left (1) and

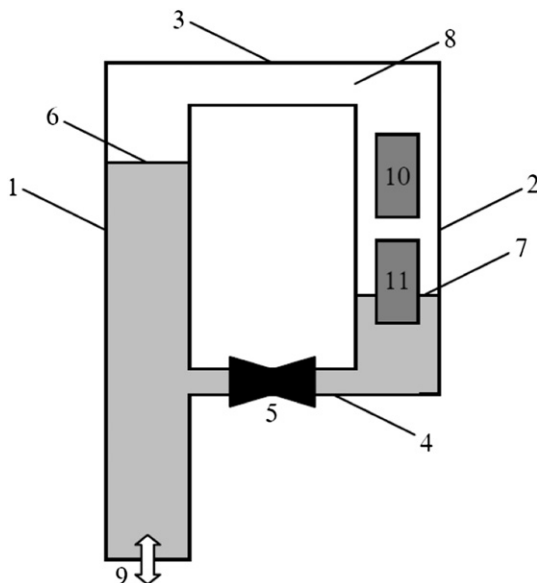


Fig. 1. Schematic of the NIFTE. The area in white corresponds to the vapor volume in which the working fluid is in the vapor phase, while the area in gray corresponds to the liquid volume in which the working fluid is in the liquid phase. Component 1 is the power cylinder (denoted by subscript ‘p’ in the text) and 2 is the displacer cylinder (‘d’), 3 is the vapor connection, and, 4 and 5 are the liquid feedback connection and valve (‘f’). Levels 6 and 7 are the working fluid vapor–liquid interfaces in the power (left) and displacer (right) cylinders. The combined vapor volume 8 (above levels 6 and 7) is assumed to be an adiabatic vapor chamber (‘ad’). Interface 9 is connected to the load line (‘l’). Components 10 (hot) and 11 (cold) are heat exchanger blocks (‘th’).

the second is inside the displacer cylinder on the right (2). The total volume of vapor at the top of the NIFTE (8) includes the vapor volumes in the power and displacer cylinders, together with the volume of the horizontal “vapor connection” pipe (3). This combined space is assumed to be an “adiabatic vapor chamber” (8). The second horizontal pipe that connects the power and displacer cylinders further down is referred to as the “liquid feedback connection” (4) and contains a valve termed the “feedback valve” (5). The power cylinder (1) transmits the fluid displacement to the load (9) below it. We treat the case where the working fluid and power transmitting fluid are the same.

The displacer cylinder (2) contains two heat exchanger blocks arranged vertically, namely the hot (10) and cold (11) heat exchangers. The heat transfer coefficient associated with single phase convection is much lower (typically by orders of magnitude) than that associated with phase change, and hence we neglect all modes of heat transfer other than evaporation and condensation. Evaporation occurs when working fluid in the liquid phase comes into contact with the surface of the hot heat exchanger (10), whereas condensation occurs when working fluid in the vapor phase comes into contact with the cold heat exchanger (11). The latter is the dominant heat transfer process at the instant portrayed in Fig. 1, when the vapor–liquid interface (7) is at the height of the cold heat exchanger (11).

Starting from the instant of condensation depicted in the figure, a reduction of mass of working fluid in the vapor phase (white) leads to a decrease in pressure in the vapor volume of the NIFTE (i.e. the combined space at the top of the power and displacer cylinders (8), and in the vapor connection). This reduction in pressure causes level 6 to be forced upwards in the power cylinder (1), and fluid to be sucked upwards into the NIFTE from the load line (9). As level 6 rises in the power cylinder (1), the hydrostatic pressure difference between the power (1) and displacer (2) cylinders first decelerates and then reverses the upwards motion of level 6. At this crucial point in the operation of the NIFTE the feedback connection (4) allows liquid to flow from the power (1) to the displacer cylinder (2) that contains the heat exchanger blocks. This, in conjunction with the reduced pressure in the vapor phase (8), causes level 7 to rise, bringing liquid into contact with the hot heat exchanger (10). The evaporation of this liquid increases volume of vapor (white) and leads to pressurization of the vapor volume that forces level 6 downwards, while fluid is displaced downwards and out of the NIFTE into the load line (9). As level 6 falls, the hydrostatic pressure difference between the power (1) and displacer (2) cylinders first decelerates and then reverses the downwards motion of level 6. The feedback connection (4) now allows liquid to flow from the displacer (2) to the power (1), while the pressurization causes level 7 to descend onto the cold heat exchanger whence condensation can re-commence, completing one cycle of the NIFTE. The process then repeats itself. The role of the feedback valve (5) is, importantly, to regulate the flow rate through the feedback connection (4), which determines the rate of liquid exchange between the power (1) and displacer (2) cylinders, and consequently the oscillation frequency of the entire device. The oscillating fluid displacement into and out of the load line (9) can be harnessed in various ways, one of which involves the use of a pair of check valves in opposite directions and in parallel to each other in order to achieve an uni-directional flow for pumping [3]. A linear approximation of the operation of the NIFTE can be described by an LRC circuit, that is, an electrical circuit with inductors (L), resistors (R) and capacitors (C), as explained in the following section.

2.3. Spatially lumped and one-dimensional linear dynamic model

In Smith [1–3] a simple, but powerful model for the dynamic behavior of the NIFTE was proposed, following Backhaus and Swift [4], Ceperley [7] and Huang and Chuang [15]. The model of the

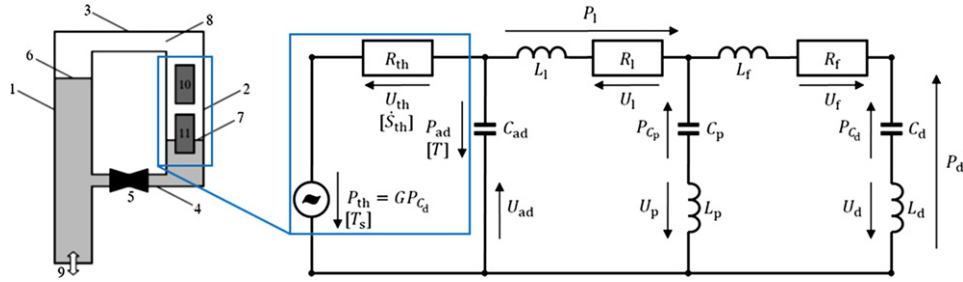


Fig. 2. NIFTE model circuit diagram, where G is the feedback gain, R_i is a resistance, C_i is a capacitance, L_i is an inductance, P_i is a pressure and U_i is a volumetric flow rate. Subscript 'th' denotes the thermal domain (heat exchanger blocks 10 and 11 in the repeat of Fig. 1 on the left), 'ad' the adiabatic domain (vapor volume 8, above liquid levels 6 and 7), 'l' the load (below 9), 'p' and 'd' the power (1) and displacer (2) cylinders, and 'f' the feedback connection: pipe and valve (4,5).

NIFTE involved a modification to the earlier models by these authors to incorporate heat flows with irreversible heat transfer, and an extension that allowed a description of phase change processes. A description of this model is presented in Ref. [3], however, the essential elements are reproduced in a more condensed form below, with some additional insights.

The model involved lumped (spatially averaged and thus independent) and first-order linearized sub-models for each component of the NIFTE that were interconnected to derive a complete model for the whole device. The linearization allows analogies to be drawn with analog electrical components, thus enabling an electrical network to be constructed for the NIFTE. The network contains resistors (accounting for viscosity, fluid drag and thermal resistance) and capacitors (for gravity and compressibility). Here we extend this approach, with the added inclusion of inductors to capture finite inertial effects in line with Solanki et al. [21].

In more detail, the dominant physical process undergone in each sub-component is firstly identified and modeled linearly. By drawing analogies between the physical variables of pressure (P) and voltage (E), volumetric flow rate (U) and current (I), temperature (T) and voltage (E), and entropy flow rate ($\dot{S}$) and current (I), the linear, lumped equations involving the raw thermal-fluid variables are transformed into a suitable network of electrical elements. The use of voltage to represent both pressure and temperature, and the use of current to represent both volumetric flow rate and entropy flow rate necessitates the definition of coupling equations, if both are to be used within a single model.

Referring to the electrical network in Fig. 2, resistances to heat or fluid flow are represented by a resistors (R); hydrostatic pressure (i.e. gravitational potential energy) of the liquid and adiabatic compressibility of the vapor are both represented by capacitors (C); and fluid inertia is represented by inductors (L). The dynamic equation of each element is $I = E/R$, $I = C \cdot \dot{E}$ and $\dot{I} = E/L$, respectively, where E is the potential difference across the component and I is the current through it. The physical quantities used to evaluate the value of each electrical component in Fig. 2 are shown in Table 1, with an explanation of all variables in the caption.

Note that the network in Fig. 2 represents a model of connected interacting processes, and that some of these are thermal (heat) flow processes while others are fluid flow processes. Now, the rate of change of pressure can be written as,

$$\frac{dP_{ad}}{dt} = \dot{S}_{th} \left(\frac{\partial P_{ad}}{\partial S_{th}} \right)_{V_{ad}} - U_{ad} \left(\frac{\partial P_{ad}}{\partial V_{ad}} \right)_{S_{th}} \quad (5)$$

The first term is determined purely by heat transfer processes and the second by adiabatic ones. For a two-phase process, it can be shown without approximation from the Clausius–Clapeyron equation, and by noting that PV^γ is constant for a reversible adiabatic (isentropic) process, that this is equivalent to,

$$\frac{dP_{ad}}{dt} = \frac{h_{fg}}{\langle c_v \rangle v_{fg}} \dot{S}_{th} + \frac{\gamma P_{ad}}{V_{ad}} U_{ad} \quad (6)$$

where $\langle c_v \rangle$ is the mass-averaged specific heat capacity at constant volume of a volume of liquid–vapor mixture in equilibrium. With reference to the second term on the right-hand side (RHS) of Eq. (6), consider the adiabatic compression/expansion process undergone by the working fluid vapor in the vapor chamber (8 in Fig. 1) due to the time-varying changes to the liquid levels in the power (6) and displacer (7) cylinders. This is a 'fluid' process and as such can be purely described by changes in a pressure (P) and a volumetric flow rate (U). In this case, pressure is the 'potential' (or 'voltage') and the volumetric flow rate is the 'current'. Let P_{ad} , V_{ad} be the time-varying pressure and volume of the vapor, such that the process can be described by $P_{ad} V_{ad}^\gamma = \text{const}$. For small perturbations of pressure and volume around an equilibrium $P_{ao} = P_o$, V_{ao} we have,

$$\begin{aligned} \frac{1}{P_{ao}} \frac{dP_{ad}}{dt} + \frac{\gamma}{V_{ao}} \frac{dV_{ad}}{dt} &= 0 \\ \Rightarrow U_{ad}(t) &= \frac{V_{ao}}{\gamma P_{ao}} \frac{dP_{ad}(t)}{dt} = \frac{V_{ao}}{\gamma P_{ao}} \dot{P}_{ad}(t) \text{ or } \tilde{U}_{ad}(S) \\ &= \frac{V_{ao}}{\gamma P_{ao}} s \tilde{P}_{ad}(s), \end{aligned} \quad (7)$$

Table 1

Expressions for the components appearing in the NIFTE as modeled in Fig. 2. The subscript 'fg' refers to phase change and 'o' to a time-averaged value; ΔS_{fg} and Δv_{fg} denote the specific entropy and specific volume changes associated with phase change; k is a relevant thermal conductance; T_o and $P_o = P_{ao}$ are the time-averaged temperature and pressure; V_{ao} is the mean total volume in the vapor phase; μ and ρ are the viscosity and density of the liquids; and l_i and d_i are pipe lengths and diameters.

Thermal-Fluid Effect	Electrical Component Analogy	Expression
Heat exchanger thermal resistance	Resistor	$R_{th} = (s_{fg}/v_{fg})^2 T_o/k$
Feedback valve & Load flow drag (viscous/pressure)	Resistor	$R_l = 128 \mu l_1 / \pi d_1^4$ & $R_f = 128 \mu l_f / \pi d_f^4$
Power & Displacer cylinder hydrostatic pressure	Capacitor	$C_p = \pi d_p^2 / 4 \rho g$ & $C_d = \pi d_d^2 / 4 \rho g$
Adiabatic vapor chamber compressibility	Capacitor	$C_{ad} = V_{ao} / \gamma P_{ao}$
Power & Displacer cylinder inertia (liquid mass)	Inductor	$L_p = 4 \rho l_p / \pi d_p^2$ & $L_d = 4 \rho l_d / \pi d_d^2$
Feedback connection & Load inertia (liquid mass)	Inductor	$L_f = 4 \rho l_f / \pi d_f^2$ & $L_l = 4 \rho l_l / \pi d_l^2$

that behaves equivalently to a capacitor $I = C_{ad} \cdot \dot{E}$, with $C_{ad} = V_{ao}/\gamma P_{ao}$. Note that here the tilde signifies the Laplace transform of the function, e.g. $\tilde{U}_{ad} = \mathcal{L}\{U_{ad}\}$, and s is the Laplace variable.

In another example, not considered by Smith [1–3], we assume that the fluid flow in the load (9) is akin to viscous laminar flow in a smooth pipe (i.e. at low Reynolds numbers Re) and at low frequencies (i.e. at low Womersley parameters Wo) when the flow is quasi-steady. Let P_1 be the pressure difference across the load and U_1 the volumetric flow rate through it. In the absence of inertia ($L_1 = 0$ in Fig. 2) P_1 acts only across R_1 , and the flow resistance (i.e. dimensionless friction factor/drag coefficient c_d) is modeled as,

$$c_d = \frac{P_1}{\frac{1}{2} \rho u_1^2} \frac{l_1}{d_1} = \frac{64}{Re_d} \Rightarrow U_1 = \frac{1}{128 \mu l_1 / \pi d_1^4} P_1, \quad (8)$$

that resembles a resistor $I = 1/R_1 \cdot E$, with $R_1 = 128 \mu l_1 / \pi d_1^4$. Here, $u_1 = U_1 / (\pi d_1^2 / 4)$ is the bulk velocity in the load. At higher frequencies one can no longer neglect inertia, which is then included in the form of an inductor L_1 in Fig. 2. This component is sufficient to account for changes in the amplitude and the phase of P_1 relative to U_1 , as demonstrated successfully in Ref. [22].

Now, consider instead the section indicated by the enclosed box in Fig. 2. This represents the heat exchangers (10 and 11), and specifically, the process of two-phase heat transfer by which the periodic lapping of the hot/cold heat exchanger blocks by the working fluid liquid level (7) in the displacer cylinder (2) drives an alternating process of generation/condensation of vapor directly into/out of the adiabatic vapor chamber (8). This is a ‘thermal’ process that can be described by changes in temperature (T) and entropy flow rate ($\dot{S}$), whereby the ‘potential’ (or ‘voltage’) is now the temperature and the ‘current’ is the entropy flow rate. To first-order this process is described by,

$$\dot{S}_{th} = \frac{1}{T_o/k} (T_s - T), \quad (9)$$

where k is the thermal conductance (inverse of the thermal resistance) between the solid block and the working fluid, and T_o is the mean boiling point temperature over which the working fluid evaporates and condenses.

It should be stated that for the purposes of our linearized model $T_o = T_{sat}$ is the saturation temperature that corresponds to a saturated vapor pressure of the working fluid equal to the time-averaged pressure in the adiabatic vapor chamber $\bar{P}_{ad} = P_{ao} = P_{sat}$. It is also equal to the mean temperature between the hot source and cold sink, as well as the time-averaged working fluid temperature in the vapor phase within the adiabatic vapor chamber $T_o = T_{sat} = \bar{T}_s = \bar{T}$.

Returning to the section indicated by the enclosed box in Fig. 2, we note that the thermal process variables must be coupled to (or ‘referred to’) the fluid variables in the rest of the circuit. For this, we require a pair of coupling equations. The first one is for the flow variable,

$$\dot{S}_{th} = \dot{m}_{th} \frac{\langle c_v \rangle}{T} \Delta T + \dot{m}_{th} s_{fg}, \quad (10)$$

where $\dot{m}_{th}$ the mass flow rate of fluid over the solid boundary that changes phase, $\langle c_v \rangle$ is a suitably averaged specific heat capacity at constant volume of the liquid–vapor mixture, ΔT is the total temperature change of the liquid–vapor mixture including excursions outside the saturation region, and s_{fg} the specific entropy change during evaporation. Given the large density change associated with phase change, the net mass flow away from the solid surface can be re-expressed as $\dot{m}_{th} = U_{th}/v_{fg}$. For small oscillations about P_{ao} , T_o in the saturation region of the working fluid, and if we

assume that $T_o s_{fg} \gg \langle c_v \rangle \Delta T$ such that $\langle c_v \rangle \frac{\Delta T}{T_o} + s_{fg} \approx s_{fg}$ (correct to about 97% for water/steam and 87% for pentane at typical NIFTE pressure amplitudes), we obtain,

$$\dot{S}_{th} = \frac{s_{fg}}{v_{fg}} U_{th}. \quad (11)$$

The second coupling relation concerns the two potential variables. Continuing with the same assumption of thermal processes dominated by phase change used above for the flow variable, we have,

$$T_s = \left(\frac{\partial T}{\partial P} \right)_{sat} P_{th}; \quad T = \left(\frac{\partial T}{\partial P} \right)_{sat} P_{ad}, \quad (12)$$

where $(\partial T / \partial P)_{sat}$ is the rate of change of temperature with pressure in the saturation region. We note further to Eq. (10) that after neglecting bulk kinetic and potential energy changes, energy conservation requires that,

$$\begin{aligned} \dot{Q} &= \dot{H} \Rightarrow T \dot{S}_{th} = P_{ad} U_{th} + \dot{m}_{th} \langle c_v \rangle \Delta T \\ \Rightarrow \dot{S}_{th} &= \frac{P_{ad}}{T} U_{th} + \frac{\langle c_v \rangle}{v_{fg}} \frac{\Delta T}{T} U_{th}. \end{aligned} \quad (13)$$

The first term on the RHS of Eq. (13) represents the displacement work advected away from the solid–fluid interface and the second term represents the internal energy advected from the interface. Eqs. (10) and (13) can be combined to give $(\partial T / \partial P)_{sat} = v_{fg} / s_{fg}$, which is correct to >99% for pentane at typical NIFTE pressure amplitudes.

Having derived the coupling equations we return to Eq. (9), which describes the process of two-phase heat transfer in the heat exchangers. This process can now be re-written as,

$$U_{th} = \frac{\left(\frac{\partial T}{\partial P} \right)_{sat}}{\frac{s_{fg}}{v_{fg}} T_o / k} (P_{th} - P_{ad}) = \frac{1}{\left(s_{fg} / v_{fg} \right)^2 T_o / k} (P_{th} - P_{ad}), \quad (14)$$

and the behavior can be modeled by a resistor $I = 1/R_{th} \cdot E$, with $R_{th} = (s_{fg} / v_{fg})^2 T_o / k$.

Results concerning all components in Fig. 2 are summarized in Table 1. Finally, it should be stated that the choice of all thermal–fluid–electrical analogies has been made to enforce the condition that electrical power, which can only be dissipated in resistors, is directly equivalent to either: (1) a rate of thermal exergy dissipated in the thermal domain due to irreversible heat transfer $I \cdot \Delta E = U \cdot \Delta P = \dot{S} \cdot \Delta T = \dot{X}$ (here, U and ΔP are the thermal quantities $\dot{S}$ and ΔT referred to the fluid domain); or (2) a rate of exergy dissipated in the fluid domain $I \cdot \Delta E = U \cdot \Delta P = \dot{W}$, either by conversion to useful work done on a load, or by conversion to lost work in a parasitic component.

2.4. Solution procedure

Similarly to Ref. [21], a ‘nominal’ value of each electrical parameter in Fig. 2 was calculated in the first instance based on the expressions in Table 1. The nominal value was obtained from the known geometric design variables, the solid properties of the selected construction materials, and the fluid properties of the selected working fluid from the actual NIFTE prototype described in detail in Ref. [3]. From this combination of nominal parameters, we proceeded to perform a parametric investigation in which each R , C and L parameter was varied independently inside a certain range, while the rest were kept at their nominal value. The ranges for each parameter were selected so as to describe a reasonable incremental design perturbation from the existing NIFTE prototype, always

leading to an acceptable and realistic configuration that we deem as practically achievable in the short term. Table 2 shows the nominal values and investigated ranges of each parameter.

Since all aforementioned components are assumed linear, the complete NIFTE network is also linear. For simplicity, but without loss of generality, we treat the case of sinusoidal oscillations with a single angular frequency $\omega = 2\pi f$ in all quantities. We limit ourselves to considering the performance (efficiency) of the NIFTE at marginal stability, i.e. in conditions in which it exhibits continuous oscillations of constant amplitude with minimal gain. The internal feedback process with gain G , which can be seen on the far left in Fig. 2, is increased until this marginal stability condition is established. The gain is related directly to the spatial temperature gradient established in the source/sink heat exchanger configuration, and thus the difference between the hot and cold temperatures available externally to the device and the design of the heat exchanger configuration. In a real NIFTE a gain setting higher than that at marginal stability will not result in an unstable system, as the system will not be able to sustain continually increasing oscillations and will eventually saturate in a complex non-linear limit cycle behavior. However, this is a sub-optimal setting from the point of view that it demands a greater temperature difference between the heat source and the heat sink (e.g. a higher heat source temperature) than that required at the marginal stability condition. Thus, the marginal stability set-point can be interpreted as the lowest temperature difference condition for which a given NIFTE configuration will achieve continuous oscillations.

At this point we note the relative amplitudes of oscillation and (relative) phases of all variables. These are then used in order to evaluate the integrals for the heat input rate $\dot{Q}_{in}$, the external heat supply rate $\dot{Q}_h$, the net power output $\dot{W}$ and net exergy flow rate $\dot{X}$ in Eqs. (1–4), which for single-frequency sinusoidal signals simplify to:

$$\dot{Q}_{in} = \omega T_0 \hat{S} / \pi + \omega \hat{T} \hat{S} \cos \angle \{T, \hat{S}\} / 4 \quad (15)$$

$$\dot{Q}_h = \omega T_0 \hat{S} / \pi + \omega \hat{T}_s \hat{S} \cos \angle \{T_s, \hat{S}\} / 4 \quad (16)$$

$$\dot{W} = \hat{P} \hat{U} \cos \angle \{P, U\} / 2 \quad (17)$$

$$\dot{X} = \omega \hat{T}_s \hat{S} \cos \angle \{T_s, \hat{S}\} / 2 \quad (18)$$

where $\hat{T}$ and $\hat{T}_s$ are the temperature amplitudes of the main cycle (i.e. the working fluid) and its thermal environment (measured at the internal surface of the source/sink heat exchangers) respectively, $\hat{S} = \Delta S / 2$ is the entropy amplitude of the main cycle such that $\dot{S} = \omega \hat{S}$ is the entropy flow amplitude, and $\angle \{a, b\}$ is the phase angle between variables a and b .

Table 2

Investigated values of all electrical parameters in Fig. 2 and Table 1. Showing both nominal values that correspond to the actual NIFTE prototype described in detail in Ref. [3]; and the full investigated range of each parameter.

Electrical Parameter	Symbol and Definition	Value		Units
		Nominal	Investigated Range	
Power cylinder hydrostatic capacitance	$C_p = \pi d_p^2 / 4 \rho g$	7.43×10^{-8}	3.22×10^{-9} to 3.22×10^{-7}	$m^4 s^2 / kg$
Power cylinder liquid mass inductance	$L_p = 4 \rho l_p / \pi d_p^2$	3.77×10^5	3.16×10^4 to 3.16×10^7	kg / m^4
Displacer cylinder hydrostatic capacitance	$C_d = \pi d_d^2 / 4 \rho g$	7.35×10^{-8}	4.31×10^{-8} to 2.58×10^{-7}	$m^4 s^2 / kg$
Displacer cylinder liquid mass inductance	$L_d = 4 \rho l_d / \pi d_d^2$	1.80×10^5	9.96×10^4 to 1.39×10^6	kg / m^4
Feedback connection fluid flow resistance	$R_f = 128 \mu l_f / \pi d_f^4$	2.13×10^6	1.64×10^3 to 1.33×10^9	$kg / m^4 s$
Feedback connection liquid mass inductance	$L_f = 4 \rho l_f / \pi d_f^2$	4.74×10^6	4.39×10^4 to 3.95×10^8	kg / m^4
Heat exchanger thermal resistance	$R_{th} = (s_{fg} / v_{fg})^2 T_0 / k$	5.02×10^8	2.84×10^8 to 1.77×10^9	$kg / m^4 s$
Adiabatic vapor chamber compressibility	$C_{ad} = V_{ao} / \gamma P_{ao}$	1.76×10^{-9}	6.40×10^{-10} to 1.84×10^{-8}	$m^4 s^2 / kg$
Load fluid flow resistance	$R_l = 128 \mu l_l / \pi d_l^4$	4.08×10^6	4.08×10^2 to 5.04×10^8	$kg / m^4 s$
Load liquid mass inductance	$L_l = 4 \rho l_l / \pi d_l^2$	1.27×10^7	1.27×10^3 to 4.24×10^9	kg / m^4

Finally, from the quantities in Eqs. (16–18) we can evaluate all required efficiencies defined previously. Hence, for the exergetic efficiency of the working fluid $\eta_{ex,wfl}$ we obtain the expression,

$$\eta_{ex,wfl} = \frac{\dot{W}_{wfl}}{\dot{X}} = \frac{\hat{P}_{ad}}{\hat{P}_{th}} \frac{\cos \angle \{P_{ad}, U_{th}\}}{\cos \angle \{P_{th}, U_{th}\}}, \quad (19)$$

while for the exergetic efficiency of the overall device $\eta_{ex,dev}$ we have [21],

$$\eta_{ex,dev} = \frac{\dot{W}_{dev}}{\dot{X}} = \frac{\hat{P}_1}{\hat{P}_{th}} \frac{\hat{U}_1}{\hat{U}_{th}} \frac{\cos \angle \{P_1, U_1\}}{\cos \angle \{P_{th}, U_{th}\}}. \quad (20)$$

Similarly, the thermal efficiency of the working fluid $\eta_{th,wfl}$ is given by,

$$\eta_{th,wfl} = \frac{\dot{W}_{wfl}}{\dot{Q}_h} = \frac{\dot{W}_{wfl}}{\dot{X}} \left(\frac{\dot{Q}_h}{\dot{X}} \right)^{-1} = \frac{\hat{P}_{ad}}{\hat{P}_{th}} \cos \angle \{P_{ad}, U_{th}\} \times \left(\frac{2/\pi}{\hat{T}_s/T_0} + \frac{1}{2} \cos \angle \{P_{th}, U_{th}\} \right)^{-1}, \quad (21)$$

and the thermal efficiency of the overall device $\eta_{th,dev}$ by,

$$\eta_{th,dev} = \frac{\dot{W}_{dev}}{\dot{Q}_h} = \frac{\dot{W}_{dev}}{\dot{X}} \left(\frac{\dot{Q}_h}{\dot{X}} \right)^{-1} = \frac{\hat{P}_1}{\hat{P}_{th}} \frac{\hat{U}_1}{\hat{U}_{th}} \cos \angle \{P_1, U_1\} \times \left(\frac{2/\pi}{\hat{T}_s/T_0} + \frac{1}{2} \cos \angle \{P_{th}, U_{th}\} \right)^{-1}. \quad (22)$$

In the above expressions we have used the results $\angle \{T_s, \hat{S}\} = \angle \{P_{th}, U_{th}\}$ and $\angle \{T, \hat{S}\} = \angle \{P_{ad}, U_{th}\}$. Note that even though we have made the assumption that the heat transfer processes between the heat exchangers and the working fluid, which are responsible for driving heat into and out of the cycle, are governed exclusively by pure dissipative phenomena in such a way that $\dot{Q} \propto \Delta T$, and even though (consequently) there can be no phase shift between $\hat{S}$ and $(T_p - T)$, there is still a phase difference between $\angle \{T_p, \hat{S}\}$ and $\angle \{T, \hat{S}\}$. This point and its relation to the performance of the NIFTE is investigated closely in Section 3.6. In addition, for the two thermal efficiencies η_{th} we have used $\hat{T}_s/T_0 = 0.05$ and $\bar{T}_s = T_0$, i.e. a 30 K peak difference between the hot source and cold sink at $T_0 = 300$ K. The maximum Carnot efficiency, based on hot and cold temperatures of 315 and 285 K respectively, is 9.5%.

3. Results and discussion

This section considers the thermal efficiency of the NIFTE, defined as $\eta_{th} = \dot{W} / \dot{Q}_h$, and the exergetic efficiency of the NIFTE,

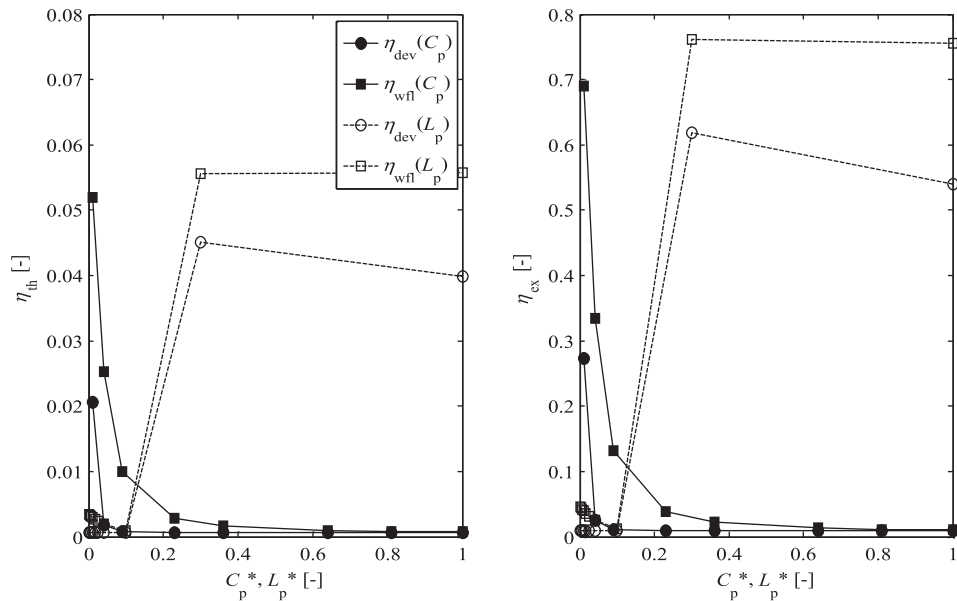


Fig. 3. Effect of power cylinder capacitance C_p and inductance L_p on the NIFTE device and working fluid thermal efficiencies, $\eta_{th,dev}$ and $\eta_{th,wfl}$ respectively (left); and device and working fluid exergetic efficiencies, $\eta_{ex,dev}$ and $\eta_{ex,wfl}$ respectively (right). The investigated parameters were varied one at a time, with all others set to a nominal value. Abscissas normalized by the largest value in the investigated range.

defined as $\eta_{ex} = \dot{W}/\dot{X}$, which were evaluated from the expressions in Eqs. (19)–(22). As previously stated, efficiencies based on the assumption that fluid-mechanical dissipation in the feedback valve component is negligible (as in Refs. [1–3]) are referred to as working fluid efficiencies η_{wfl} and describe the efficiency of the thermodynamic cycle undergone by the working fluid; whereas efficiencies which account for the fluid-mechanical parasitic dissipation in the feedback connection are referred to as device efficiencies η_{dev} and describe the efficiency of the overall device. Based on these definitions, results are presented in Figs. 3–8 for the

effects of selected design parameters (R_s , C_s , L_s in Fig. 2 and Table 1) on the NIFTE efficiencies:

- (i) Thermal efficiency of the device $\eta_{th,dev}$;
- (ii) Thermal efficiency of the working fluid system $\eta_{th,wfl}$;
- (iii) Exergetic efficiency of the device $\eta_{ex,dev}$; and,
- (iv) Exergetic efficiency of the working fluid $\eta_{ex,wfl}$.

In all plots, the parameter under investigation was varied independently, with all other parameters set to a nominal value.

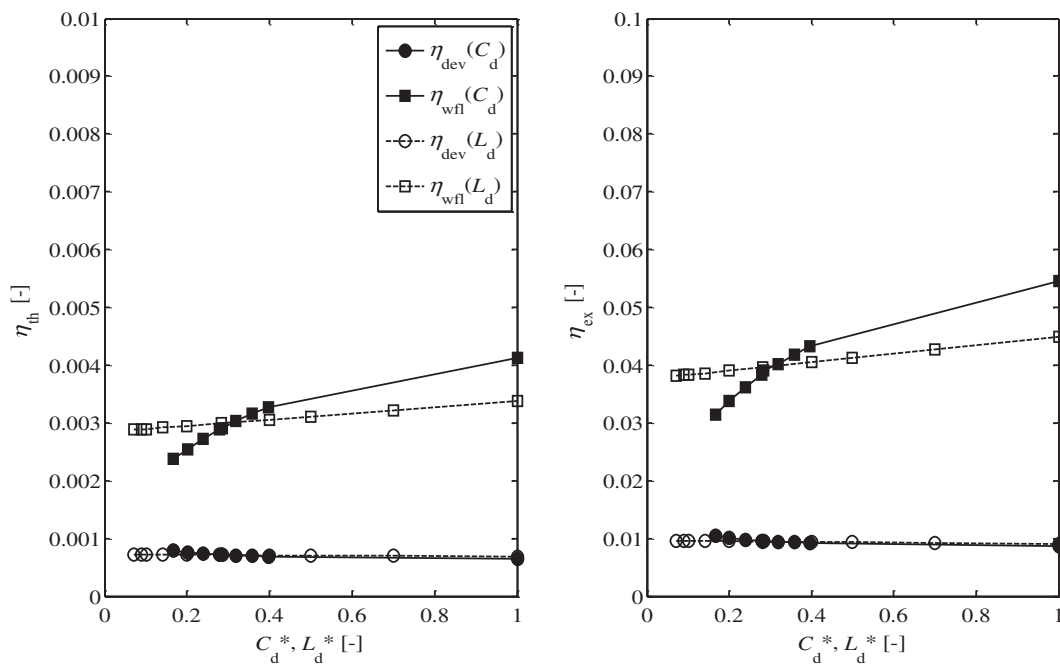


Fig. 4. Effect of displacer cylinder capacitance C_d and inductance L_d on the NIFTE device and working fluid thermal efficiencies, $\eta_{th,dev}$ and $\eta_{th,wfl}$ respectively (left); and device and working fluid exergetic efficiencies, $\eta_{ex,dev}$ and $\eta_{ex,wfl}$ respectively (right). The investigated parameters were varied one at a time, with all others set to a nominal value. Abscissas normalized by the largest value in the investigated range.

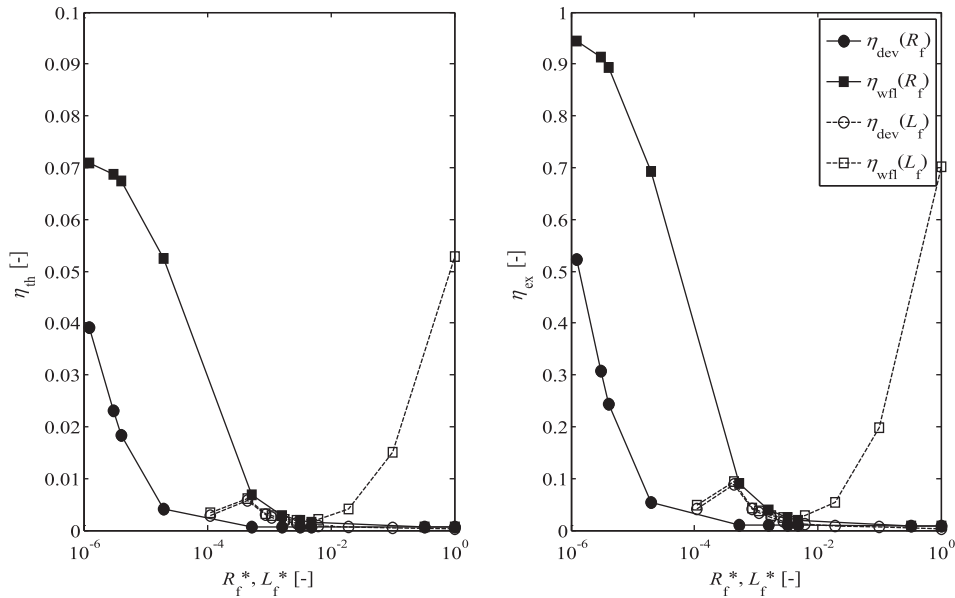


Fig. 5. Effect of feedback valve resistance R_f and connection inductance L_f on the NIFTE device and working fluid thermal efficiencies, $\eta_{th,dev}$ and $\eta_{th,wfl}$ respectively (left); and device and working fluid exergetic efficiencies, $\eta_{ex,dev}$ and $\eta_{ex,wfl}$ respectively (right). The investigated parameters were varied one at a time, with all others set to a nominal value. Abscissas normalized by the largest value in the investigated range.

Specifically, Fig. 3 shows results relating to the role of the power cylinder (component 1 in Fig. 2), Fig. 4 shows results relating to the displacer cylinder (2 in Fig. 2), Fig. 5 shows results relating to the feedback connection (4 and 5 in Fig. 2), and Fig. 6 shows results relating to the heat exchangers (10 and 11 in Fig. 2) and the adiabatic vapor chamber (the total white region 8 in Fig. 2). The abscissas (on the horizontal axis) are normalized by the largest value in the investigated range. All efficiency values obtained from perturbations of the NIFTE from a nominal design can be compared to the efficiencies of the nominal NIFTE (i.e. corresponding to the actual NIFTE prototype described in Ref. [3]), which are summarized in Table 3.

By means of comparison, estimated values of $\eta_{ex,dev}$ from experiments on a NIFTE water pump [3] with a 125–150 W heater

sustaining a temperature of 90 °C and a cooling water temperature of 4 °C were in the range 0.4–1.7%, while $\eta_{ex,wfl}$ was between 3.7 and 9.5%. Based on these source and sink temperatures the Carnot efficiency is 24%, which corresponds to $\eta_{th,dev} = 0.1$ –0.4% and $\eta_{th,wfl} = 0.9$ –2.3%. Even though these reported ranges cover configurations that extend beyond nominal settings, they provide some confidence that the values provided in Table 3 are consistent with realistic NIFTE performance characteristics and a valid starting point for this current analysis.

Finally, Figs. 7 and 8 contain results of a sensitivity analysis that was performed in order to identify the key parameters that control the efficiencies of the NIFTE. Here, the abscissa values of impedance on the horizontal axes have been normalized by each variable's nominal value.

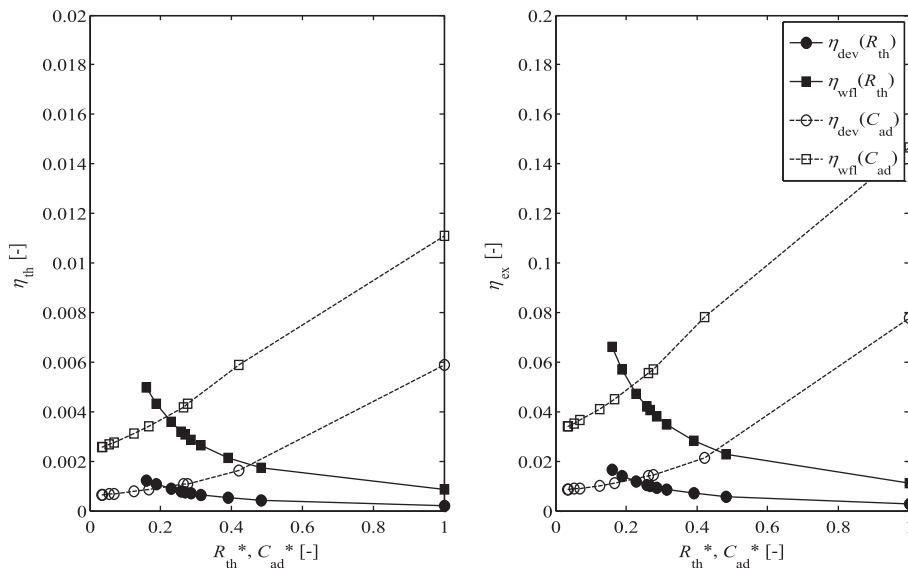


Fig. 6. Effect of thermal resistance R_{th} and adiabatic capacitance C_{ad} on the NIFTE device and working fluid thermal efficiencies, $\eta_{th,dev}$ and $\eta_{th,wfl}$ respectively (left); and device and working fluid exergetic efficiencies, $\eta_{ex,dev}$ and $\eta_{ex,wfl}$ respectively (right). The investigated parameters were varied one at a time, with all others set to a nominal value. Abscissas normalized by the largest value in the investigated range.

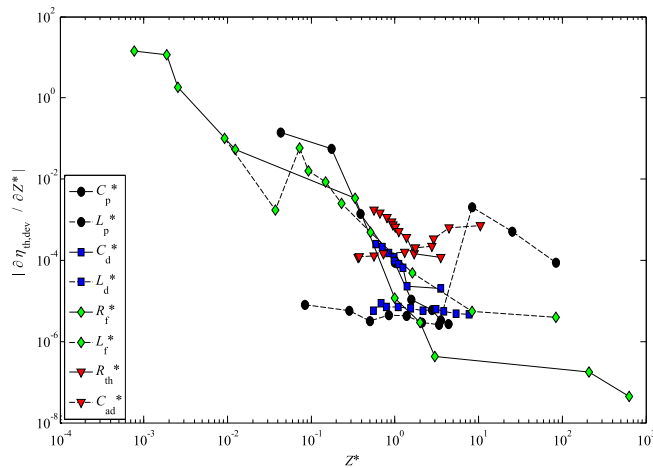


Fig. 7. Change in the NIFTE device thermal efficiency $\eta_{th,dev}$ due to change in: hydrostatic pressure capacitance C_p and liquid mass inductance in the power cylinder L_p ; hydrostatic pressure capacitance C_d and liquid mass inductance in the displacer cylinder L_d ; fluid flow resistance R_f and liquid mass inductance L_f in the feedback connection; thermal resistance in the heat exchangers R_{th} ; and, adiabatic vapor chamber capacitance C_{ad} . Abscissa values of impedance Z^* on the horizontal axes have been normalized by each variable's nominal value.

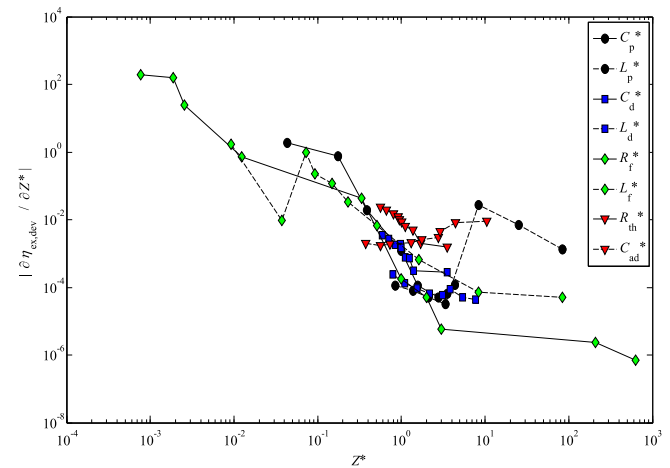


Fig. 8. Change in the NIFTE device exergetic efficiency $\eta_{ex,dev}$ due to change in: hydrostatic pressure capacitance C_p and liquid mass inductance in the power cylinder L_p ; hydrostatic pressure capacitance C_d and liquid mass inductance in the displacer cylinder L_d ; fluid flow resistance R_f and liquid mass inductance L_f in the feedback connection; thermal resistance in the heat exchangers R_{th} ; and, adiabatic vapor chamber capacitance C_{ad} . Abscissa values of impedance Z^* on the horizontal axes have been normalized by each variable's nominal value.

3.1. Power cylinder

Variations in the thermal efficiency η_{th} (left) and exergetic efficiency η_{ex} (right) due to changes in the geometry of the power cylinder are shown in Fig. 3. The maximum achievable efficiency values obtained by making these changes are $\eta_{th} = 5.6\%$ and $\eta_{ex} = 76\%$. These are considerably higher than the values stated in Table 3 for nominal NIFTE design and suggest that step improvements are possible. Recall that the capacitance and inductance of the power cylinder are given by the expressions $C_p = \pi d_p^2 / 4\rho g$ and $L_p = 4\rho l_p / \pi d_p^2$, respectively. Hence:

- An increase in the diameter of the power cylinder d_p would cause C_p to increase and L_p to decrease;
- An increase in the liquid density of the working fluid ρ would cause C_p to decrease and L_p to increase; and,
- An increase in the *time-averaged* length of the liquid column in the power cylinder l_p (related to the time-average of level 6 in Fig. 2) would cause L_p to increase, but would have no effect on C_p .

In Fig. 3 we note that the two types of device efficiency (thermal and exergetic) lead to consistent design interpretations. For example, both η_{th} and η_{ex} experience a sharp drop as L_p is reduced below 0.3, and a sharp rise as C_p is reduced below 0.2. This similarity can be also observed Figs. 4–6 that relate to the displacer cylinder, feedback valve, heat exchangers and adiabatic vapor volume. However, the device efficiency is not necessarily optimal when the working fluid cycle exhibits its maximum efficiency. For example, looking at the figure on the left, η_{dev} is relatively insensitive to changes in C_p except at extremely low values (lower than about 0.05) where it shows an improvement, while η_{wfl} increases considerably as C_p is gradually reduced below 0.2. This contradiction will be even more evident in Fig. 4 that relates to the effect of the displacer cylinder.

From Fig. 3 it can be seen that C_p and L_p can be used to make significant improvements to the two device efficiencies, $\eta_{th,dev}$ and $\eta_{ex,dev}$. In absolute terms, C_p can be used to increase $\eta_{th,dev}$ by up to 2.0% and $\eta_{ex,dev}$ by up to 26%, while L_p can be used to increase $\eta_{th,dev}$ by up to 4.4% and $\eta_{ex,dev}$ by up to 61%. From this observation it

becomes evident that the power cylinder (i.e. the choice of C_p and L_p) is an important component in determining the overall efficiency of the NIFTE. Specifically, the results imply that improved NIFTE efficiencies can be achieved by reducing C_p and increasing L_p , at least within the range of investigate conditions (and given the associated assumptions). Both of these improvements can be realized simultaneously by decreasing the diameter of the power cylinder d_p and increasing the liquid density of the working fluid ρ . An additional increase in the length of the liquid column in the power cylinder l_p would act to further increase L_p , without affecting C_p .

Changes to C_p and L_p can also considerably affect the two working fluid efficiencies, $\eta_{th,wfl}$ and $\eta_{ex,wfl}$. Specifically, $\eta_{th,wfl}$ showed a 5.1% variation over the investigated range of C_p , and a 5.5% variation over the investigated range of L_p . Similarly, over the same ranges $\eta_{ex,wfl}$ varied by 68% and 75%, respectively.

3.2. Displacer cylinder

Equivalent plots to those in Fig. 3, but for the displacer cylinder are shown in Fig. 4. The capacitance and inductance of the displacer cylinder are given by $C_d = \pi d_d^2 / 4\rho g$ and $L_d = 4\rho l_d / \pi d_d^2$, respectively, such that:

- An increase in the diameter of the displacer cylinder d_d causes C_d to increase and L_d to decrease;
- An increase in the liquid density of the working fluid ρ causes C_d to decrease and L_d to increase; and,
- An increase in the *time-averaged* length of the liquid column in the displacer cylinder l_d (related to the time-average of level 7 in Fig. 2) causes L_d to increase, but would have no effect on C_d .

Table 3

Nominal device and working fluid exergetic and thermal efficiencies corresponding to the actual NIFTE prototype.

Efficiency Definition	Value (%)
$\eta_{th,dev}$	0.1
$\eta_{th,wfl}$	0.3
$\eta_{ex,dev}$	1.0
$\eta_{ex,wfl}$	3.9

From Fig. 4 it is observed that the magnitude of η_{dev} is 3–6 times smaller than that of η_{wfl} , or in absolute terms lower by 0.2–4.6%. Moreover, the device efficiencies η_{dev} are insensitive to the design of the displacer cylinder (i.e. the choice of C_d and L_d), with $\eta_{\text{th,dev}} \sim 0.07\%$ and $\eta_{\text{ex,dev}} \sim 0.9\%$ (to within ± 1 s.f.) over the entire investigated range. On the other hand η_{wfl} is approximately doubled by increasing C_d within the same range. Still, this doubling corresponds to a 0.2% absolute increase in $\eta_{\text{th,wfl}}$ and an 18% absolute increase in $\eta_{\text{ex,wfl}}$, which is considerably less than the improvement achievable with the careful design of the power cylinder suggested previously.

It is important to recognize that changes to the displacer cylinder parameters (C_d and L_d) can have contradictory results on the magnitude, but also the trends of the NIFTE's efficiency, depending on the choice of efficiency definition. This highlights the fact that the efficiency of the thermodynamic cycle undergone by the working fluid η_{wfl} can diverge significantly from the overall efficiency of the NIFTE device η_{dev} , which includes the dissipation of useful power in the parasitic feedback valve that is external to the main thermodynamic cycle. By extension, it demonstrates that neglecting this dissipated power, as was assumed in Refs. [1–3] in order to allow a more simple analysis and interpretation of the NIFTE, is not generally valid unless the feedback resistance R_f is extremely large, in which case the gain required in order to attain marginally stable oscillations is also very high. In this work we choose η_{dev} as a more representative indication of the overall capabilities of the NIFTE. On this basis, we will focus our discussion from this point onwards on the overall device efficiency η_{dev} , though results will also be presented for the working fluid efficiency η_{wfl} . We conclude that the displacer cylinder is not important in determining the efficiency of the NIFTE.

3.3. Feedback connection

Plots concerning the feedback connection are shown in Fig. 5. The maximum achievable efficiency values obtained by making changes to the feedback connection variables are $\eta_{\text{th}} = 7.1\%$ and $\eta_{\text{ex}} = 94\%$. As with the power cylinder, these values are significantly higher than those obtained for the nominal NIFTE design (see Table 3), which again implies that the current design can be vastly improved with careful design. By comparison, typical thermal efficiencies of around 3–4% were reported for dry free-liquid-piston Stirling (Fluidyne) engines [8–11,23,24], though figures as high as 7% were reported for some larger units. The resistance and inductance of the feedback connection are given by $R_f = 128 \mu l_f / \pi d_f^4$ and $L_f = 4 \rho l_f / \pi d_f^2$, so:

- An increase in the length of the feedback connection l_f causes both R_f and L_f to increase;
- An increase in the diameter of the feedback connection d_f causes both R_f and L_f to decrease;
- An increase in the liquid dynamic viscosity of the working fluid μ causes R_f to increase, but has no effect on L_f ; and,
- An increase in the liquid density of the working fluid ρ causes L_f to increase, but has no effect on R_f .

The feedback connection, like the power cylinder, has an important role to play in determining the eventual efficiency of the NIFTE. An examination of the effects of the parameters R_f and L_f on our selected efficiency definition η_{dev} (i.e. only the circles) in Fig. 5, reveals that changes to R_f have a much greater ability to influence η_{dev} than changes to L_f . In particular, Fig. 5 (left) shows that R_f (filled circles) can be used to improve $\eta_{\text{th,dev}}$ by up to 3.9% in absolute terms, and (right) to improve $\eta_{\text{ex,dev}}$ by up to 51% in absolute terms.

The corresponding values for L_f (hollow circles) are 0.6% and 9%, respectively. Thus, the sensitivities of $\eta_{\text{th,dev}}$ and $\eta_{\text{ex,dev}}$ to R_f are about 6 times greater than their sensitivities to L_f . The sensitivities of the important device efficiency definitions η_{dev} to all NIFTE parameters will be examined in detail in the discussion relating to Figs. 7 and 8.

So the model predicts that an efficiency improvement can be achieved by reducing R_f , and to a lesser extent L_f . Both of these can be realized simultaneously by decreasing the length l_f and increasing the diameter d_f of the feedback connection. An additional decrease in the liquid dynamic viscosity of the working fluid μ and a decrease in the liquid density of the working fluid ρ will also act to independently lessen R_f and L_f , respectively.

Recall however, from the sub-section on the power cylinder optimization, that an increased liquid density ρ was required there for maximum efficiency. Hence, the optimization of the NIFTE with respect to this variable is more complex. One approach to avoid this conflict in requirements is to set R_f and L_f by designing the geometry of the feedback connection (l_f and d_f), and to set C_p and L_p independently by designing the geometry of the power cylinder (d_p), without the need to change the working fluid.

3.4. Heat exchangers and adiabatic vapor chamber

Finally, we turn to the heat exchangers and the total vapor volume at the top of the NIFTE. Plots concerning the effect of these two components on the efficiency of the NIFTE are shown in Fig. 6. The heat transfer process in the heat exchangers is described by the thermal resistance $R_{\text{th}} = (s_{\text{fg}}/\nu_{\text{fg}})^2 T_0/k$, while the adiabatic capacitance associated with the adiabatic vapor chamber is $C_{\text{ad}} = V_{\text{ao}}/\gamma P_{\text{ao}}$. From these expressions, an increase in R_{th} can be established with either a:

- Higher specific enthalpy due to phase change s_{fg} ; or,
- Lower specific volume due to phase change ν_{fg} ; or,
- Higher time-averaged (saturation) temperature T_0 ; or,
- Lower thermal conductance k .

In addition, C_{ad} can be increased with either a:

- Higher mean total volume in the vapor phase V_{ao} ; or,
- Higher ratio of specific heat capacities of the working fluid in the vapor phase γ ; or,
- Lower time-averaged (saturation) vapor pressure P_{ao} , which is directly linked to the (saturation) temperature T_0 .

As before, we examine the effects of the two independent variables (here R_{th} and C_{ad}) on our selected (device) efficiency definition η_{dev} (i.e. only the circles) in Fig. 6. The thermal resistance R_{th} (filled circles) has a less noticeable effect on η_{dev} than that of C_{ad} (hollow circles). In fact, in Fig. 6 the sensitivities of both $\eta_{\text{th,dev}}$ and $\eta_{\text{ex,dev}}$ to C_{ad} are approximately 5 times greater than their sensitivities to R_{th} , so the first priority in maximizing η_{dev} should be to maximize C_{ad} . In more detail, decreasing R_{th} within the investigated range increases $\eta_{\text{th,dev}}$ by 0.1% (or a factor of about 2), and increasing C_{ad} increases $\eta_{\text{th,dev}}$ by 0.5% (or a factor close to 5). Similarly, decreasing R_{th} increases $\eta_{\text{ex,dev}}$ by 1.4%, and increasing C_{ad} increases $\eta_{\text{ex,dev}}$ by 6.9%.

The selection of the working fluid is an important, but complicated issue in itself and depends on a number of additional factors (not contained in the current study) that must be weighed before a decision can be made. Such factors include for example its boiling point in relation to the hot T_h and cold T_c temperatures, amongst others. Specifically with respect to the boiling point, one basic requirement is that this must be somewhere between T_h and T_c if

the working fluid is going to evaporate and condense, otherwise the NIFTE will not be able to operate.

Given that there is little scope in affecting the specific heat capacity ratio of the working fluid in the vapor phase γ , it is only possible to establish considerably larger values of C_{ad} by increasing the total volume in the vapor phase V_{ao} , and/or decreasing the time-averaged (saturation) pressure P_{ao} . With respect to the former, the total adiabatic vapor chamber volume V_{ao} can be increased by using power and displacer cylinders with larger diameters and lengths, as well as vapor connection pipes with larger diameters and lengths. Yet, the requirement concerning the power cylinder length is in partial contradiction with the condition that was established in the power cylinder sub-section for maximum efficiency, i.e. that a narrow (small d_p) power cylinder ought to be preferred. One way to overcome this problem is to use a long, but small diameter power cylinder, thus achieving both small C_p and large C_{ad} .

In contrast to V_{ao} , the designer has little control over the time-averaged (saturation) pressure P_{ao} . This pressure appears as an external boundary condition observed by the NIFTE, as it is determined by the time-mean pressure in the load to which the NIFTE is connected. In the linear model employed here the time-averaged temperature T_o that is found half-way between the source and sink temperatures, which also coincides with saturation conditions. Since P_{ao} is coupled to T_o , the time-averaged pressure P_{ao} also determines the temperatures in the heat exchangers. In fact, we expect that lower P_{ao} will be associated with lower heat exchanger temperatures. Nevertheless, we expect that C_{ad} will be higher under these conditions, leading to improved efficiencies. A lower T_o will also help to decrease R_{th} that can have a secondary beneficial effect on the NIFTE efficiency. Note however, that the linear analysis performed here is too simplistic to capture non-linear effects that become increasingly important at higher P_{ao} and higher heat exchanger temperatures.

3.5. Sensitivity analysis

In this section we re-examine the data presented previously, but in the form of a sensitivity analysis in order to identify the R , C and L parameters (and NIFTE variables) that most strongly affect the efficiencies of the NIFTE. We focus here only on the overall device efficiencies η_{dev} , which include parasitic fluid power dissipation due to viscous and pressure drag in the feedback connection (due to R_{th}). Throughout this section we assume that the designer has no control over the load which the device interfaces with, as this is set externally by the application. We therefore set the load to its nominal value and exclude all load parameters from consideration.

Fig. 7 (with key values summarized in Table 4) contains plots of the changes in the thermal efficiency of the device $\eta_{th,dev}$ due to changes in all independent device parameters, including: the capacitance due to hydrostatic pressure C_p and inductance due to liquid mass (inertia) in the power cylinder L_p ; the capacitance due to hydrostatic pressure C_d and inductance due to liquid mass (inertia) in the displacer cylinder L_d ; the resistance due to fluid flow (drag) R_f and inductance due to liquid mass (inertia) L_f in the feedback connection; the thermal resistance in the heat exchangers R_{th} ; and, the capacitance associated with compressibility of the adiabatic vapor chamber C_{ad} .

Fig. 8 (with related key results summarized in Table 5) is identical to Fig. 7, but focuses on changes to the *exergetic* efficiency of the device $\eta_{ex,dev}$. It is possible to conclude from Figs. 7 and 8, as well as the accompanying Tables 4 and 5, that the efficiencies of NIFTE are most sensitive to changes in the feedback connection (both R_f and L_f), followed by the power cylinder (C_p and L_p), and to a lesser extent the thermal resistance in the heat exchangers R_{th} and the capacitance of the adiabatic vapor chamber C_{ad} . Based on this information it becomes evident that the selection of the feedback valve as the natural operating point tuning component is the right approach.

3.6. Temperature-entropy diagrams

It has been shown in the previous sections that the feedback connection and power cylinder are important components, whose optimization can lead to significant improvements in efficiency with respect to an existing (nominal) NIFTE prototype. For instance, a decrease in R_f resulted in an improvement in $\eta_{th,dev}$ from a nominal 0.1% up to 3.9% in absolute terms, and similarly in $\eta_{ex,dev}$ from a nominal 1% up to 52%. In another example, a decrease in C_p was capable of increasing $\eta_{th,dev}$ from a nominal 0.1% to 2.1%, and $\eta_{ex,dev}$ from a nominal 1% to 27%. Corresponding working fluid efficiencies also showed considerable improvements, with $\eta_{th,wf}$ increasing from 0.3% to 7.1% at the optimal R_f and 5.2% at the optimal C_p , respectively; and with $\eta_{ex,wf}$ increasing from 4% to 94% at the optimal R_f and 69% at the optimal C_p setting.

In order to gain some insight into the underlying reasons for these vast potential improvements, it is useful to examine the main (internally reversible) thermodynamic cycle undergone by the working fluid on a T – S diagram and also to compare this with the greater area that represents the full (thermally irreversible) cycle as experienced at the external heat source and sink. As explained previously, the distinction between these two rests on the choice of temperature, either T or T_s , and the area between the two

Table 4

Device thermal efficiency $\eta_{th,dev}$ sensitivity coefficients corresponding to Fig. 7. Showing both nominal coefficients that correspond to the actual NIFTE prototype described in detail in Ref. [3]; and the maximum (of the modulus) of each coefficient over the full investigated range of each parameter, as presented in Table 2.

Parameter	Symbol and Definition	Device Efficiency Sensitivity Coefficient	
		Nominal	Maximum (Modulus)
Power cylinder hydrostatic capacitance	$C_p = \pi d_p^2 / 4 \rho g$	2.92×10^{-3}	1.85×10^{-1} [°]
Power cylinder liquid mass inductance	$L_p = 4 \rho l_p / \pi d_p^2$	3.05×10^{-4}	2.26×10^{-3}
Displacer cylinder hydrostatic capacitance	$C_d = \pi d_d^2 / 4 \rho g$	4.06×10^{-4}	4.21×10^{-4}
Displacer cylinder liquid mass inductance	$L_d = 4 \rho l_d / \pi d_d^2$	2.66×10^{-5}	4.00×10^{-5}
Feedback connection fluid flow resistance	$R_f = 128 \mu l_f / \pi d_f^4$	1.70×10^{-3}	$1.45 \times 10^{+1}$ [°]
Feedback connection liquid mass inductance	$L_f = 4 \rho l_f / \pi d_f^2$	7.41×10^{-4}	1.45×10^{-1} [°]
Heat exchanger thermal resistance	$R_{th} = (s_{fg} / v_{fg})^2 T_o / k$	2.63×10^{-3}	3.46×10^{-3}
Adiabatic vapor chamber compressibility	$C_{ad} = V_{ao} / \gamma P_{ao}$	9.65×10^{-4}	3.32×10^{-3}

^a Primary control component major variable.

^b Primary control component minor variable.

^c Secondary control component main variable.

Table 5

Device exergetic efficiency $\eta_{ex,dev}$ sensitivity coefficients corresponding to Fig. 8. Showing both nominal coefficients that correspond to the actual NIFTE prototype described in detail in Ref. [3]; and the maximum (of the modulus) of each coefficient over the full investigated range of each parameter, as presented in Table 2.

Parameter	Symbol and Definition	Sensitivity Coefficient	
		Nominal	Maximum
Power cylinder hydrostatic capacitance	$C_p = \pi d_p^2 / 4 \rho g$	1.19×10^{-3}	1.91×10^0 [°]
Power cylinder liquid mass inductance	$L_p = 4 \rho l_p / \pi d_p^2$	1.05×10^{-4}	2.79×10^{-2}
Displacer cylinder hydrostatic capacitance	$C_d = \pi d_d^2 / 4 \rho g$	1.46×10^{-3}	3.41×10^{-3}
Displacer cylinder liquid mass inductance	$L_d = 4 \rho l_d / \pi d_d^2$	1.68×10^{-4}	2.47×10^{-4}
Feedback connection fluid flow resistance	$R_f = 128 \mu l_f / \pi d_f^4$	2.02×10^{-4}	$1.93 \times 10^{+2}$ [°]
Feedback connection liquid mass inductance	$L_f = 4 \rho l_f / \pi d_f^2$	3.97×10^{-3}	1.66×10^0 [°]
Heat exchanger thermal resistance	$R_{th} = (s_{fg}/v_{fg})^2 T_o/k$	8.89×10^{-3}	2.38×10^{-2}
Adiabatic vapor chamber compressibility	$C_{ad} = V_{ao}/\gamma P_{ao}$	1.97×10^{-3}	9.26×10^{-3}

^a Primary control component major variable.

^b Primary control component minor variable.

^c Secondary control component main variable.

represents a power loss (exergy destruction) through irreversible heat transfer that occurs in R_{th} between the source/sink and the working fluid. Fig. 9 shows results from this investigation for the effects of R_f (left) and C_p (right) on the T – S (solid lines, filled squares) and T_s – S (dotted lines, hollow squares) NIFTE diagrams undergone in the relative to the nominal configuration (circles).

The areas within the two ovals are a direct indication of the efficiencies of the corresponding cycle. As expected, it can be observed in Fig. 9 that the T_s – S diagrams (hollow symbols) always enclose the T – S diagrams (filled symbols). It can also be seen that the T_s – S diagrams cover the whole vertical range (height) in the graphs T_s that corresponds to the amplitude of temperature oscillations in the heat exchangers $\hat{T}_s = 15$ K, again as expected. The nominal design however has a greatly reduced working fluid temperature oscillation amplitude $\hat{T} = 1$ K, revealing a substantial exergy loss in the thermal resistance R_{th} . This helps to explain the relatively low $\eta_{ex,wfl}$ and $\eta_{ex,dev}$, since both of these efficiency measures are adversely affected by these exergetic losses.

Continuing, the most important observation that can be made based on the plots in Fig. 9 concerns the effects on the T – S diagrams (solid lines, square symbols) of optimizing either the value of R_f (left) or C_p (right). Compared to the nominal NIFTE, the optimized designs are associated with much larger amplitudes in the working fluid temperature oscillations. Specifically, we can see clearly from these plots that the underlying effect of the parametric optimization process is to increase $\hat{T}$ to 14 K when R_f is optimized and to 10 K when C_p is optimized. This corresponds directly to the higher η_{ex} in the optimal designs relative to the nominal NIFTE. In addition, the larger relative increase in $\hat{T}$ that can be generated by varying R_f

rather than C_p within their perturbation envelopes can explain the higher η_{ex} of the R_f optimal NIFTE design. These values are summarized in Table 6.

As a final comment, it can be observed in Table 6 that unlike the nominal NIFTE, the optimized designs are capable of keeping the phase between $\angle\{T_s, \hat{S}\}$ and $\angle\{T, \hat{S}\}$ low (see Eqs. (19)–(22)). This phasing is an important parameter in determining the eventual efficiency as it controls (together with the amplitudes $\hat{T}_s$ and $\hat{T}$) the area of the Lissajous ovals in Fig. 9, with zero phase corresponding to maximum efficiency and 90° phase corresponding to zero efficiency. However, close inspection of various other sub-optimal designs (not shown here) reveals even lower phases than those reported in Table 6, suggesting that the primary reason for efficiency improvement at lower R_f and C_p is the increase in temperature amplitude ratios, rather than a decrease in the phase difference between $\angle\{T_s, \hat{S}\}$ and $\angle\{T, \hat{S}\}$.

Table 6

Result of optimization of the NIFTE with respect to (w.r.t.) R_f and C_p in terms of external and working fluid temperature amplitudes ($\hat{T}_s$ and $\hat{T}$) and corresponding phases ($\angle\{T_s, \hat{S}\}$ and $\angle\{T, \hat{S}\}$). Comparison with the nominal design.

Variable	Nominal Value	Optimized NIFTE	
		w.r.t. R_f	w.r.t. C_p
$\hat{T}_s$ (K)	15	15	15
$\hat{T}$ (K)	1	14	10
$\angle\{T_s, \hat{S}\}$ (deg.)	3°	8°	4°
$\angle\{T, \hat{S}\}$ (deg.)	52°	8°	6°

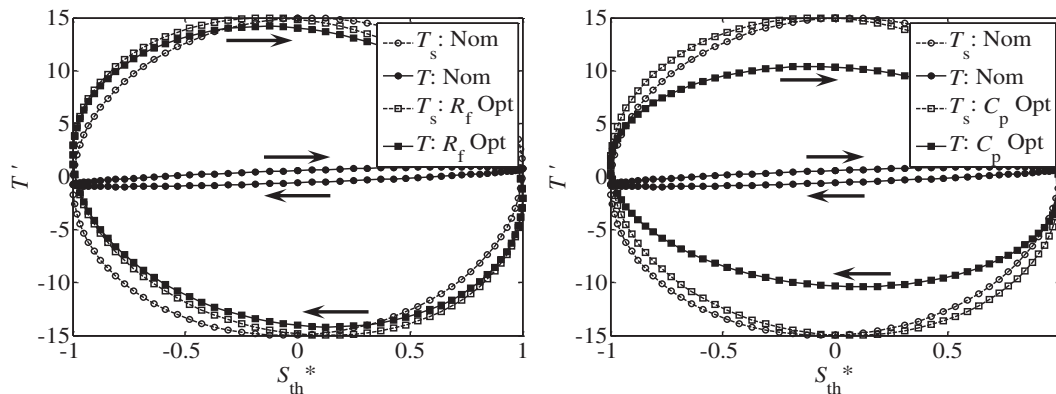


Fig. 9. First-order linear approximation phase plane plots (Lissajous ovals) of the NIFTE thermodynamic cycle, showing the nominal configuration (circles) with $R_f = 2.13 \times 10^6 \text{ kg m}^{-4} \text{ s}^{-1}$ and $C_p = 7.43 \times 10^{-8} \text{ m}^4 \text{ s}^2 \text{ kg}^{-1}$, as well as optimized cycles with respect to R_f ($=1.64 \times 10^3 \text{ kg m}^{-4} \text{ s}^{-1}$) with C_p fixed at its nominal value (left, square symbols) and C_p ($=3.22 \times 10^{-9} \text{ m}^4 \text{ s}^2 \text{ kg}^{-1}$) with R_f fixed at its nominal value (right, square symbols). Each figure includes both working fluid T – S (solid lines, filled symbols) and device T_s – S (dotted lines, hollow symbols) diagrams. Abscissa values of entropy fluctuations S_{th}^* on the horizontal axes are normalized by their amplitude.

4. Conclusions

Starting from a first-order linear dynamic model of the NIFTE that consists of a network of interconnected spatially lumped components, we investigated parametrically the effect of various component variables (geometric and other) on the thermal (first law) and exergetic (second law) efficiencies of this device. Results from the parametric optimization process in which the parameters were varied independently within a specified range, were then compared to efficiency values associated with a nominal NIFTE configuration, based on values that were estimated from an existing prototype. Great improvements were demonstrated relative to the nominal NIFTE design, implying that the careful design can have a significant effect on the performance of the final device.

The study highlighted critical components that require careful optimization, namely the feedback connection valve, the power cylinder, the adiabatic volume and the thermal resistance in the heat exchangers. Relative to the nominal NIFTE, an efficient design would feature a lower feedback connection valve resistance, with a shorter connection length and larger connection diameter; a smaller diameter but taller power cylinder; a larger (time-mean) combined vapor volume at the top part of the device; and improved heat transfer behavior (i.e. reduced thermal resistance) in the hot and cold heat exchanger blocks.

The effect of these modifications was to increase the exergetic efficiency of the device by 50% points, which also corresponds to a 3.8% point increase in thermal efficiency. The NIFTE was modeled as operating across a temperature difference between heat source and sink that one would associate with low grade heat (30 K). Given a maximum theoretical (Carnot) efficiency associated with these temperatures of 9.5%, the reported improvement is considered noteworthy. Close inspection of T – S diagrams revealed that the underlying reason for the improvement was an increase in the working fluid temperature amplitude relative to that made available to it by the heat source and sink, as it undergoes the thermodynamic cycle.

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